

# An improved adaptive switching control based on quasi-ARX neural network for nonlinear systems

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**Abstract** In this paper, an improved switching mechanism based on quasi-linear auto regressive exogenous (quasi-ARX) neural network (QARXNN) is presented for the adaptive control of nonlinear systems. The proposed switching control is composed of a QARXNN-based prediction model and an improved switching mechanism using two new adaptive control laws, first is moving average filter law and second is new switching law. Since the control result of nonlinear predictor is better than the linear predictor in most of the time, the adaptive control with a simple switching mechanism has many useless switching during the processing. Hence, the improved smooth switching mechanism is proposed to replace the original switching mechanism; it can improve the performance by reducing the useless switching while guaranteeing stability of the system control. The simulations show that the efficiency of the proposed control method is satisfied in stability, improve control accuracy and robustness.

**Keywords** Adaptive switching control · An improved switching mechanism · Quasi-ARX neural network prediction model

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## 1 Introduction

Adaptive control of nonlinear dynamical systems has attracted much attention and developed significantly during the last of few decades. Many adaptive control methods have been proposed and the corresponding stability and convergence have been proved [1, 2]. Unfortunately, the performance of linear control models cannot satisfy the requirement. Hence, some nonlinear prediction models have been developed for nonlinear systems to overcome the difficulty in design of predictor and controller for nonlinear systems. One approach to identify and control nonlinear dynamical systems is neural networks because of its ability to approximate the arbitrary mapping to any desired accuracy [3, 4, 5].

However, there are two major problems on those neural network models. The first, their parameters do not have useful interpretations. The second, they do not have a friendly interface for controller design and system analysis. To solve these problems, in the previous work, a quasi-linear auto regressive exogenous (quasi-ARX) neural network (QARXNN) modeling scheme has been proposed based on well-developed linear system theory and can be extended to nonlinear systems. The models consist of two parts: a macro model part and a kernel part [6, 7, 8]. The QARXNN model has two properties: the linear property and the nonlinear property. Based on the model characteristics, two controllers can be obtained: one linear controller and one nonlinear controller. The linear controller is used to ensure the control stability and the nonlinear controller is utilized to improve the control accuracy. The 0/1 switching mechanism is proposed between the two controllers. If the switching flag is 0, then a linear controller is employed otherwise a nonlinear controller is employed. In such a way, the quasi-ARX prediction model uses only one model to achieve function of two or more models.

Nevertheless, there is still aspects needed to be improved in the above control method that the 0/1 hard switching method is not very smooth. With the hard switching law used, the linear and nonlinear predictors are alternately used, while from the 0/1 switching control result, it is obvious that the switching control does not need to switch to 0 in most of the time, so a smooth switching mechanism is constructed based on the system switching criterion function [9]. Furthermore, the original switching mechanism has some useless switching problems which influence control accuracy. To solve the problems and to make the switching process much more smooth, the improved switching mechanism contained two new methods: one is to reduce the fluctuation of switching parameters, and the other is to design the new switching law proposed in this paper.

This paper is organized as follows. Section 2 briefly describes the problem to be solved and the improved QARXNN prediction model is introduced based on a neural network using an improved switching mechanism, then the parameter identification methods are given. Section 3 discusses the stability of the adaptive control with the improved switching mechanism. Section 4 provides the numerical simulations to demonstrate the effectiveness of the proposed method, and Sect. 5 presents the conclusions.

## 2 Statement of the problem

Considering a single-input-single-output (SISO) nonlinear time-invariant dynamical system with input output relation described as follow,

$$y(t+d) = g(\varphi(t)) \quad (1)$$

$$\varphi(t) = [y(t+d-1) \cdots y(t+d-n) u(t) \cdots u(t-m+1)]^T \quad (2)$$

where  $y(t)$  denotes the output at time  $t (t = 1, 2, \dots)$ ,  $u(t)$  the input,  $d$  the known integer time delay,  $\varphi(t)$  the regression vector, and  $n, m$  the system orders.  $g(\cdot)$  is a nonlinear function. The following assumption will be used:

**Assumption 1** (i)  $g(\cdot)$  is a continuous function, and at a small region around  $\varphi(t) = 0$ , it is  $C^\infty$  continuous. (ii) there is a reasonable unknown controller which may be expressed by  $u(t) = \tilde{\rho}(\tilde{\xi}(t))$ , where  $\tilde{\xi}(t) = [y(t) \cdots y(t-n)u(t-1) \cdots u(t-m)y^*(t+1) \cdots y^*(t+1-l)]^T$  ( $y^*(t)$  denotes reference signal). (iii) the system has a globally uniformly asymptotically stable zero dynamics. The following assumption will be used in Sect. 3 to analyze the control system stability:

**Assumption 2** (i) The linear parameter  $\theta$  lies in a compact region  $B$ . (ii) The nonlinear term is globally bounded i.e.,  $\Psi^T(t)W_2\Gamma(W_1\xi(t) + B) < \Delta$ .

Under the continuous condition, the unknown nonlinear function  $g(\varphi(t))$  can be performed Taylor expansion on a small region around  $\varphi(t) = 0$ :

$$y(t+d) = g'(0)\varphi(t) + \frac{1}{2}\varphi^T(t)g''(0)\varphi(t) \cdots \quad (3)$$

where the prime denotes differentiation with respect to  $\varphi(t)$ .

## 3 Implementation of an efficient controller

### 3.1 Quasi-ARX neural network

The elements of  $\Theta(\xi(t))$  are unknown nonlinear function of  $\xi(t)$ , which can be parameterized by neuro-fuzzy networks as in [10, 11]. Parameterizing  $\Theta_\xi$  with a MIMO neural network, the quasi-ARX prediction model is expressed by:

$$\hat{y}(t+d | t, \xi(t)) = \Psi^T(t)\aleph(\xi(t), \Omega) \quad (4)$$

where  $\aleph(\cdot, \cdot)$  is a generalized 3-layer neural network with  $n$  input nodes,  $M$  sigmoid hidden nodes and  $n+1$  linear output nodes. The 3-layer neural network can be expressed by:

$$\aleph(\xi(t), \Omega) = \theta + W_2\Gamma(W_1\xi(t) + B) \quad (5)$$

where  $\Omega = \{W_1, W_2, B, \theta\}$  are the parameters set of the neural network. The  $W_1 \in R^{M \times N}$  and  $W_2 \in R^{M \times (N+1)}$  are the weight matrices of the first and second layers, while  $B \in R^{M \times 1}$  is the bias vector of hidden nodes. The  $\theta \in R^{(N+1) \times 1}$  represents the bias vector of output nodes and  $\Gamma$  is the diagonal nonlinear operator with identical sigmoid elements  $\sigma$  (for example:  $\sigma(x) = \frac{1-e^{-x}}{1+e^{-x}}$ ). Moreover,  $\xi(t)$  is the input variables of neural network. Then, we can express the QARXNN prediction model as:

$$\hat{y}(t+d | t, \xi(t)) = \Psi^T(t)\theta + \Psi^T(t).W_2\Gamma(W_1\xi(t) + B) \quad (6)$$

The linear part  $\theta$  and nonlinear part  $W_1, W_2$ , and  $B$ . Now,  $\theta$  is updated as:

$$\hat{\theta}(t) = \hat{\theta}(t-d) + \frac{a(t)\Psi(t-d)e_1(t)}{1 + \Psi(t-d)^T\Psi(t-d)} \quad (7)$$

where  $\hat{\theta}(t)$  is the estimation of  $\theta$  at time instant  $t$  and  $a(t) = 1$  if  $|e_1(t)| > 2\Delta$  otherwise  $a(t) = 0$  where  $e_1(t)$  is error of the linear part defined by:

$$e_1(t) = y(t - d) - \Psi(t)^T \hat{\theta}(t) \tag{8}$$

The error of nonlinear part parameters is adjusted by BP algorithm and defined by:

$$e_2(t) = y(t + d) - \Psi(t)^T \hat{\theta}(t) - \Psi(t)^T \hat{W}_2(t) \Gamma (\hat{W}_1(t) \xi(t) + \hat{B}(t)) \tag{9}$$

where  $\hat{W}_1(t)$ ,  $\hat{W}_2(t)$ , and  $\hat{B}(t)$  are the estimations of  $W_1$ ,  $W_2$ , and  $B$  at time instant  $t$ , respectively.

### 3.2 Proposed controller

Consider a previous work [5], a minimum variance control with the criterion function as follows,

$$M(t + 1) = \left[ \frac{1}{2} (y(t + d) - y^*(t + d))^2 + \frac{\lambda}{2} u(t)^2 \right] \tag{10}$$

where  $\lambda$  is weighting factor for the control input. The controllers can be achieved by solving,

$$\frac{\partial M(t + 1)}{\partial u_i} = 0 \quad i = 1, 2 \tag{11}$$

By solving (11), two controllers can be derived based on QARXNN prediction model as follows,

$$C_1 : u_1(t) = \frac{\hat{b}_0^1}{\hat{b}_0^1 \hat{b}_0^1 + \lambda} ((\hat{b}_0^1 - \hat{b}^1(q^{-1})q)u(t - 1) + y^*(t + 1) - \hat{a}^1(q^{-1})y(t)) \tag{12}$$

$$C_2 : u_2(t) = \frac{\hat{\beta}_{0,t}}{\hat{\beta}_{0,t}^2 + \lambda} ((\hat{\beta}_{0,t} - \hat{\beta}(q^{-1})q)u(t - 1) + y^*(t + 1) - \hat{\alpha}(q^{-1}, \xi(t))y(t)) \tag{13}$$

where

$$\begin{aligned} \hat{a}(q^{-1}) &= \hat{a}_0 + \hat{a}_1 q^{-1} + \dots + \hat{a}_{n-1} q^{-n+1}; \\ \hat{b}(q^{-1}) &= \hat{b}_0 + \hat{b}_1 q^{-1} + \dots + \hat{b}_{m+d-2} q^{-m-d+2}; \\ \hat{\alpha}(q^{-1}, \xi(t)) &= \hat{\alpha}_{0,t} + \hat{\alpha}_{1,t} q^{-1} + \dots + \hat{\alpha}_{n-1,t} q^{-n+1}; \\ \hat{\beta}(q^{-1}, \xi(t)) &= \hat{\beta}_{0,t} + \hat{\beta}_{1,t} q^{-1} + \dots + \hat{\beta}_{m+d-2} q^{-m-d+2}; \end{aligned}$$

The coefficients can be obtained as follows,

$$\begin{aligned} [\hat{\alpha}_0 \dots \hat{\alpha}_{n-1} \hat{\beta}_0 \dots \hat{\beta}_{m+d-2}] &= \hat{\theta} \\ [\hat{\alpha}_{0,t} \dots \hat{\alpha}_{n-1,t} \hat{\beta}_{0,t} \dots \hat{\beta}_{m+d-2,t}] &= \hat{\theta} + \hat{W}_2 \Gamma (\hat{W}_1 \xi(t) + \hat{B}) \end{aligned}$$

Therefore, the switching criterion function can be expressed as:

$$J_i(t) = \sum_{l=k}^t \frac{a_i(l)(\|e_i(l)\|^2 - 4\Delta^2)}{2(1 + a_i(l)\Psi(l-k)^T P_i(l-k-1)\Psi(l-k))} + c * \sum_{l=t-N+1}^t (1 - a_i(l)\|e_i(l)\|^2), \quad i = 1, 2 \tag{14}$$

where  $N$  is an integer, and  $c \geq 0$  is a predefined constant. The previous work [7] introduces a switching parameter  $\chi(t)$  based on the criterion function  $J_1(t)$  and  $J_2(t)$ :

$$\chi(t) = \begin{cases} 1 & \text{if } x(t) > K \\ x(t) & \text{if } k \leq x(t) \leq K \\ 0 & \text{if } x(t) < k \end{cases} \tag{15}$$

where  $x(t) = \frac{J_1(t)}{J_1(t) + J_2(t)}$ ,  $K$  and  $k$  are constants which satisfy  $k \in (0, 0.5)$ ,  $K \in (0.5, 1)$ .

Now, a smooth switching controller is obtained based on the switching parameter  $\chi(t)$ , it is less (or equal to) 1.0

$$C : u(t) = (1 - \chi(t))u_1(t) + \chi(t)u_2(t) \tag{16}$$

The smooth switching law  $\chi(t)$  is calculated from input and output signals and model errors, and then it will be used in the control model.

### 3.3 Improved switching mechanism

After analyzing the control processing, some problems still exist: (1) Since the control result of nonlinear predictor is better than the linear predictor in most of the time, it is better to remain nonlinear predictor while ensuring the stability of the control to improve the accuracy. (2) By updating the linear parameters, the errors of the predictor  $e_1$  and  $e_2$  will change a lot, so the stability of control system is proved. To improve the accuracy of the control result, we may not only consider a single  $\chi(t)$ , but also the tendency of  $\chi(t)$ . In this way, we can reduce the unreasonable switching occurred by the unstable  $J_1(t)$ ,  $J_2(t)$ . (3) The switching control method is used to guarantee the stability of control system with the switching between linear and nonlinear model. However,  $J_2(t)$  is bounded in most of the time, so there are many useless switching in original smooth switching method. Motivated by the reasons mentioned before, in this paper, the improved smooth switching mechanism is designed to increase the control accuracy with two methods:

**Method 1: Moving average filter** To make the switching control more smooth, the first improving method is adding the moving average filter in the switching parameter  $\chi(t)$ , with the preset threshold parameters  $k$  and  $K$  to improve the accuracy and the adaptation in the controller by reducing the unreasonable switching in the control process.

$$\chi(t) = \begin{cases} 1 & \text{if } x(t) > K \\ \frac{1}{\sum_{i=0}^{M-1} \zeta_i} \sum_{i=0}^{M-1} \zeta_i \chi(t-1) & \text{if } k \leq x(t) \leq K \\ 0 & \text{if } x(t) < k. \end{cases} \tag{17}$$

**Method 2: New switching law** As mentioned before, to ensure the stability of the control system, the switching mechanism should consider the error of nonlinear predictor first, while the linear predictor error  $e_1(t)$  is globally bounded. Furthermore, the nonlinear predictor is bounded in most control processing.

For this condition, the switching law is designed to make the switching mechanism remain in nonlinear predictor as much as possible. The other goal of the design is to improve the control accuracy in the nonlinear predictor which has better performance than the linear predictor.

$$\chi(t) = \begin{cases} 1 & \text{if } \max(J_1(t)) \geq J_2(t) \\ 0 & \text{if } \max(J_1(t)) < J_2(t) \end{cases} \tag{18}$$

### 3.4 Stability

The stability of the two new methods are:

**Method 1: Moving average filter** Since  $J_1(t)$  is bounded, from the equation of switching function  $J_i(t)$  and the moving average filter with switching parameter  $\chi(t)$ , there exists a constant  $t_k$  such that  $\forall t > t_k, \chi(t) = 0$ , so the control error  $e(t) = e_1(t)$  is bounded with  $e_1(t)$ .

**Method 2: New switching law** Since  $J_1(t)$  is globally bounded, and  $J_2(t)$  is smaller than  $\max(J_1(t))$ , we can consider  $J_2(t)$  is bounded with  $J_1(t)$  and prove the stability same as case 1). In addition, while  $J_2(t)$  is larger than  $\max(J_1(t))$ , where  $J_2(k)$  is bounded or not, we can calculate switching parameter  $\chi(t)$  with the fuzzy switching control equation in the new switching law. This condition also exists in a constant  $t_k$  such that  $\forall t > t_k, \chi(t) = 0$ , so that the control error  $e(t) = e_1(t)$  is bounded with  $e_1(t)$ . With the improved switching mechanism, from above inequalities, the input and output of the closed-loop switching control system are bounded.

## 4 Numerical simulations

The system is nonlinear governed by:

$$y(t) = f[y(t-1), y(t-2), y(t-3), u(t-1), u(t-2)] \tag{19}$$

$$f[x_1, x_2, x_3, x_4, x_5] = \frac{x_1 x_2 x_3 x_5 (x_3 - 1) + x_4}{1 + x_2 + x_3} \tag{20}$$

In the identification, a neural network with one hidden layer and 20 hidden nodes is used and the parameters are predefined as  $n_y = 4, n_u = 3, d = 1$ . Then, the system is trained off-line by the hierarchical training algorithm. This model is used on-line as an identifier whose nonlinear part is adjusted by BP algorithm and linear part by algorithm mentioned in the above section. This model is adopted on-line as an identifier by algorithm mentioned in the above section. For identification, we record 300 input–output data sets by exciting the system with a random input sequence as well.

The reference output of this example is shown as:

$$y^*(t) = \begin{cases} 0.4493y^*(t-1) + 0.57r(t-1) & t \in [1, 100] \cup [151, 200] \\ 0.5\text{sign}(0.4493y^*(t-1) + 0.57r(t-1)) & t \in [101, 150] \end{cases} \tag{21}$$

where  $r(t) = \sin(2\pi t/25)$ , the parameters of switching criterion function are chosen to be  $c = 1.2, N = 3$ , then the control result with different method is obtained.

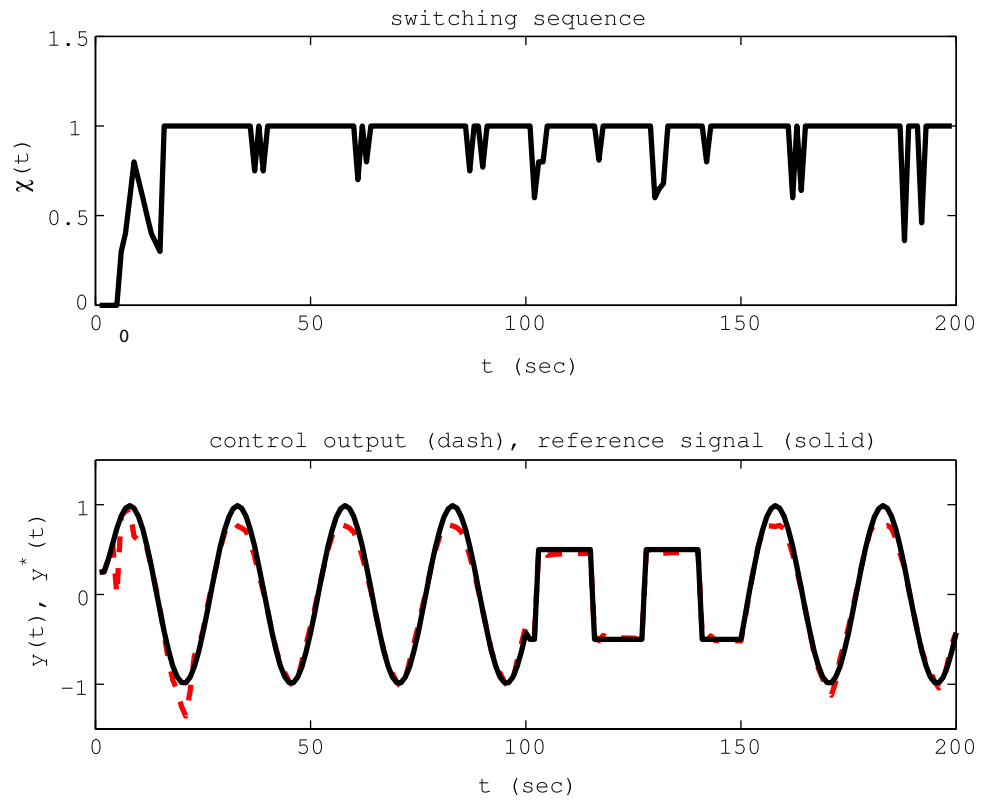
Considering the control processing, we can also find while using the original proposed switching mechanism, the switching processing is not smooth as the switching parameter  $\chi(t)$  changes a lot, while using the original smooth switching mechanism shown in Fig. 1 and while using the improved switching mechanism with the moving average filter and new switching law, where  $M = 5, [\zeta_0, \zeta_1, \zeta_2, \zeta_3, \zeta_4] = [1, 1, 1, 1, 1], k = 0.1, K = 0.9$ ; then we get the control results shown in Figs. 2 and 3. Considering the control result, we can find that using the improved smooth switching mechanism the adaptive control can reduce the useless switching with the switching parameter  $\chi(t)$  and remain in the nonlinear control as much as possible. In this way, the control processing is much smooth and improves the control performance with the reference signal.

Comparing the different control results with root mean squared error(RMSE) as follow:

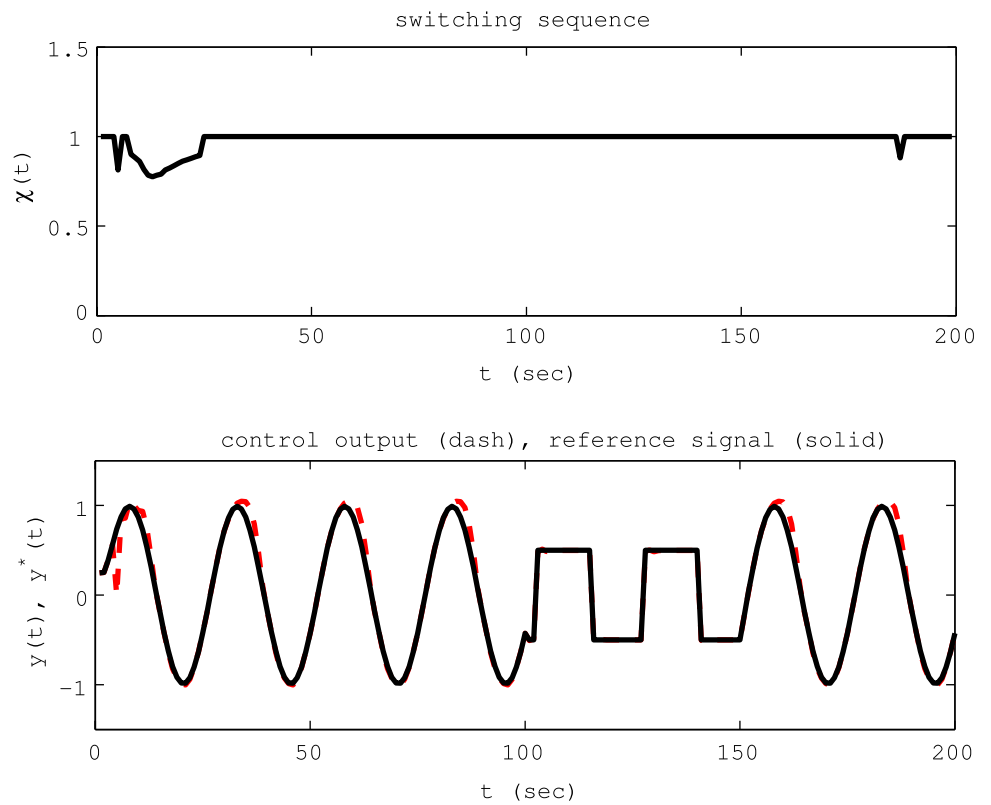
$$\text{RMSE} = \sqrt{\frac{\sum_i (y(t) - \hat{y}(t))^2}{N}} \tag{22}$$

From the control result in this Table 1, it is obvious that the improved switching mechanism can improve the control accuracy when comparing with the original method. Table 1 also gives the contrast of four methods' errors. The error of the smooth switching method is smaller than 0/1 switching method. The error of the proposed method (smooth switching with new switching law method and smooth switching with moving average filter method) is smaller than 0/1 switching method and smooth switching method. The proposed method improves the control performance.

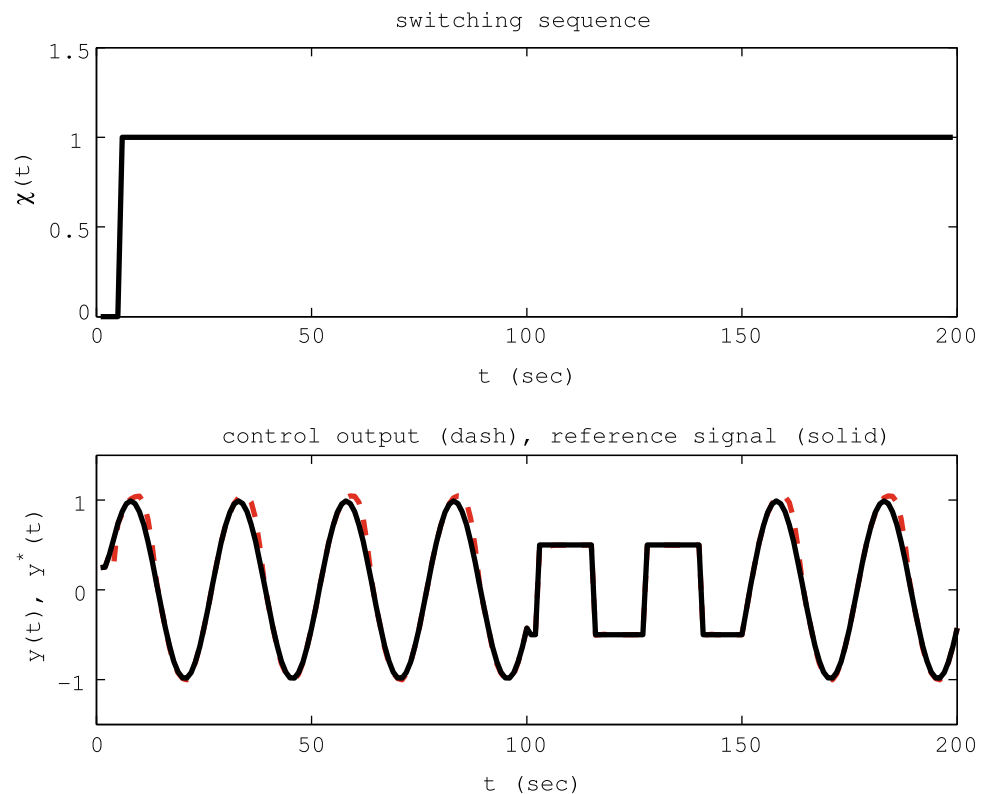
**Fig. 1** Control result of smooth switching mechanism



**Fig. 2** Control result of smooth switching mechanism with moving average filter



**Fig. 3** Control result of smooth switching mechanism with new switching law



**Table 1** Control result with RMSE

Method	RMSE
0/1 switching	0.1050
Smooth switching	0.0993
Smooth switching with moving average filter	0.0772
Smooth switching with new switching law	0.0792

## 5 Conclusions

In this paper, an adaptive control law is proposed for nonlinear dynamical systems and then the control system stability is proved. In such a way, the adaptive switching control mechanism can satisfy the stability, response and performance requirement using only one model. By the hard switching laws used, the linear and nonlinear predictors are alternately used. However, from the 0/1 switching control result, we can find that the switching control does not need to switch to 0 in most of the time so that we can design the smooth switching to improve the control result. According to the control processing, we find that the original smooth switching mechanism is not smooth enough by updating the linear and nonlinear predictor. Hence, to improve the performance of the switching control, an improved switching mechanism is designed with two methods: one improves calculation of switching parameters and

the other designs a new switching law. Consider the simulations result that the switching mechanism has been compared, we can find that the improved switching mechanism with both methods increases the control accuracy by solving the useless switching problem while ensuring the control stability.

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