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An integrated vendor–buyer replenishment policy for deteriorating items with fuzzy environment and resource constraint

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ABSTRACT

Different from previous researches, our study considers a perishable item with collaborative vendor–buyer ordering policy and finite replenishment rate. Furthermore, due to the importance of inventory and capital investment in today's fuzzy marketing environments, researches in fuzzy collaborative inventory models have become very popular research in recent decades. Therefore, in our integrated model with deteriorating inventory replenishment policy, we construct the crisp/fuzzy models with inventory investment constraints with fuzzy environments. Two different fuzzy decision-making methods are used to formulate the models. Convex fuzzy programming (CFP) method is used to maximize the weighted sum for each achievement level of the joint cost and constraints. An inverse weighted fuzzy non-linear programming (IWFNLP) is then proposed to satisfy the decision-maker's desirable achievement level of service. A heuristic algorithm with mixed-integer hybrid differential evolution (MIHDE) is developed to solve the crisp/fuzzy models. Numerical examples and sensitivity analysis are developed to investigate the effectiveness of the proposed method in the fuzzy environment. From the numerical analysis, it can be seen that the IWFNLP method is a more efficient decision-making tool than the CFP method.

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Inventory; deteriorating item; fuzzy environment; investment constraints

1. Introduction

Collaborative inventory models considering the perspectives of the vendor and the buyer have recently become very popular. Goyal (1977) was one of the first authors to develop collaborative inventory model for a single supplier–single customer

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problem. Banerjee (1986) generalized Goyal's (1977) joint economic lot-size model by assuming the vendor's finite production rate with a lot-for-lot production. Goyal (1988) extended Banerjee's (1986) model by relaxing the lot-for-lot policy and suggested that the vendor's economic production quantity should be an integer multiple of the buyer's purchase quantity. Ha and Kim (1997) further extended the concept and proposed an integrated lot-splitting model of facilitating multiple shipments in small lots. Yang and Wee (2000) applied a heuristic approach to develop an integrated vendor–buyer inventory model for deteriorating items. Yang et al. (2012) developed a constrained newsboy problem with return policy. Wee et al. (2013) developed a bi-level vendor–buyer strategies considering a time-varying product price. Alenezi et al. (2015) developed an integrated producer–retailer supply chain inventory model with imperfect production and quality. Chung et al. (2016) explored and integrated cooperative vendor–buyer inventory models in order to reduce the pressure of the increasing total cost in today's manufacturing environment. The work also considered trade credit financing and suppliers' credits. Other more detailed discussions and researches in these fields are given in Widyadana et al. (2011), Cárdenas-Barrón et al. (2012a, 2012b), Sana (2013), Teng et al. (2013), Cárdenas-Barrón et al. (2014), Chung et al. (2014), Taleizadeh et al. (2015), De and Sana (2016) and Singh and Saravanan (2017).

Even though fuzzy environment and constraints are common in practice, few researchers have devoted their studies on collaborative inventory model with fuzzy environment and constraints. A couple of fuzzy mathematical programming methods and fuzzy set theory have been developed in recent years (Bellman & Zadeh, 1970; Chen & Tsai, 2001; Ketsarapong et al., 2012; Zimmermann, 1978). Vujosevic et al. (1996) extended EOQ model by assuming fuzzy holding and ordering costs. Chang (2004) assumed a fuzzy EOQ model with imperfect quality items considered the fuzzy defective rate and fuzzy demand. Lam and Wong (1996) studied the fuzzy model for joint economic lot size problem with multiple price breaks. Roy and Maiti (1998) designed the multi-objective deteriorating inventory model with fuzzy environment using fuzzy non-linear programming. Mondal and Maiti (2003) applied genetic algorithm (GA) to solve multi-item fuzzy EOQ models. De et al. (2008) studied an EOQ model for deteriorating seasonal commodity where shortages are allowed and fuzzy cost parameters are considered in an integrated supply chain. Our study enhanced their model by considering vendor–buyer inventory investment separately and discussed a cost sharing mechanism to improve the cooperative relationship in the integrated supply chain. Wee et al. (2009) developed a multi-objective joint replenishment deteriorating inventory model with fuzzy environment. Lo (2010) developed a fuzzy integrated vendor–buyer inventory policy of deteriorating items under credibility measure. Taleizadeh et al. (2011) solved a fuzzy single period problem using meta-heuristic algorithms. Taleizadeh et al. (2013a) developed a joint single vendor–single buyer supply chain problem with stochastic demand and fuzzy lead-time, and Taleizadeh et al. (2013b) revisited a fuzzy rough EOQ model with deteriorating items by considering quantity discount and prepayment. De and Sana (2013) considered a fuzzy backorder EOQ inventory model with fuzzy shortage quantity and promotional index as decision variables. Chitra and Parvathi (2014) discussed a fuzzy

EOQ model for deteriorating items with stock dependent demand under permissible delay in payments and inflation. They did not allow for shortages. The objective is to minimize the retailers' total inventory cost assuming triangular fuzzy numbers. Das and Islam (2014) proposed a multi-objective inventory model with deteriorating items and shortage is allowed.

Later, De and Sana (2015a) discussed an alternative fuzzy EOQ inventory model with allowable backlogging and promotional sensitive demand. Pattnaik (2015) applied the concept of fuzzy Non-Linear Programming Technique to solve a single item EOQ model with budget and storage area constraints. His work assumed that demand is related to the unit price and the variation of the setup cost is followed by the order quantity. Jaggi et al. (2016) proposed a fuzzy EOQ model with delay in payments, price-dependent demand, fully shortage backlogging and different trade credit terms. Chakraborty et al. (2017) considered an integrated three-layer inventory model with non-instantaneous deteriorating item, inflation effect and delay in payments in random fuzzy environment. This integrated model in this study applied generalized reduced gradient technique to optimize total cycle time and the credit period such that it minimized the total cost. Mojaveri and Moghimi (2017) applied the graded mean integration representation (GMIR) deffuzification method in which was used for parameters deffuzification to solve the fuzzy EOQ model. In the fuzzy EOQ model, this study considers two different situations, including the related costs considered by triangular fuzzy numbers and fuzzy demand. Islam and Mandal (2017a) proposed a fuzzy EOQ model with unit production cost, time-depended holding cost, without shortages and applied two Fuzzy Geometric Programing (FGP) approaches to solve the problem. Saranya and Varadarajan (2018) considered a fuzzy EOQ model with completely backorder under acceptable shortage and Graded Mean Integration Value method is adopted to defuzzify the fuzzy total cost function. In this study, Triangular and Trapezoidal fuzzy numbers is applied to obtain the total cost. Different from the above work, Mandal and Islam (2018) considered a bell shape instead of Triangular and Trapezoidal fuzzy numbers.

Recently, many researchers and academics have considered different fuzzy environment with/without restricted Space. A few examples are: Pattnaik (2013), De et al. (2014), Das et al. (2015), De and Sana (2014, 2015b), Sen and Malakar (2015), Islam and Mandal (2017b), Sharma & Govindaluri (2018) and Chanda et al. (2018). Finally, comprehensive comparisons between the related works and our study are shown in Table 1.

Different from the above researches, our study optimizes the total joint cost of the deteriorating item vendor–buyer finite replenishment model with fuzzy environment and constraints. The fuzziness of objective and constraints in the integrated system are considered due to the variations of real market/customer demand and resource limitations. Two different fuzzy decision-making approaches are used to formulate the fuzzy models in our analysis. The first method using Convex Fuzzy Programming (CFP) is used to maximize the weighted sum for each achievement level of the fuzzy components, including the fuzzy joint cost and constraints and the second method using Inverse Weighted Fuzzy Non-linear Programming (IWFNLP) is then used to

Table 1. Comprehensive comparisons between the related works and our study.

Reference	Decisions	Deterioration item	Integrated vendor/buyer replenishment policy	With/without investment constraints	EOQ model in Fuzzy environment	Heuristic algorithm/ NLP approach	Delay in payment/ Prepayment
De et al. (2008)	• T • T_0	Yes	Yes	No	Yes	No	No
Wee et al. (2009)	• T • T_1	Yes	No	Yes	Yes	Heuristic algorithm	No
Taleizadeh et al. (2013b)	• T • T_1	Yes	No	Yes	Yes	Heuristic algorithm	Yes (Prepayment)
Chitra and Parvathi (2014)	• T • *EOQ**	Yes	No	No	Yes	No	Yes (Delay in payment)
Das and Islam (2014)	• T • T_1	Yes	No	Yes	Yes	NLP approach	No
Pattnaik (2015)	• *Demand* • *EOQ**	No	No	Yes	Yes	NLP approach	No
Jaggi et al. (2016)	• T • T_1 • *EOQ**	No	No	No	Yes	No	Yes (Delay in payment)
Chakraborty et al. (2017)	• T • T_3	Yes	Yes	No	Yes	NLP approach	Yes (Delay in payment)
Mojaveri & Moghimi (2017)	• EOQ*	No	No	No	Yes	NLP approach (Deffuzification)	No
Islam and Mandal (2017a)	• *Demand* • *Setup cost* • *EOQ**	No	No	Yes	Yes	NLP approach	No
Present paper (2018)	• n • T_2	Yes	Yes	Yes	Yes	Heuristic algorithm	No

Note: EOQ*: optimal economic order quantity; NLP: Non-Linear Programming; n = number of deliveries; T = replenishment cycle period; T_0 = shortage period; T_1 = inventory period; T_2 = Downtime period; T_3 = credit period.

satisfy the decision-maker's desirable achievement level. Different patterns of weights are assigned by the decision-maker of an enterprise for each fuzzy component such as fuzzy joint cost, fuzzy vendor's and buyer's average inventory investment constraints.

The proposed model is classified as mixed-integer nonlinear programming (MINLP) model. A heuristic algorithm with mixed-integer hybrid differential evolution (MIHDE), is developed to solve the crisp/fuzzy model. Numerical examples and sensitivity analysis are developed to investigate the effectiveness of the proposed method in the fuzzy environment. The optimum numerical results derived from the CFP and IWFNLP methods are compared and presented in tabular form.

The remainder of the paper is organized as follows. The following section briefly describes our problem definitions and introduces all notations and the mathematical formulations of the crisp and fuzzy models for the integrated vendor–buyer inventory replenishment problems. Section 3 shows how the two fuzzy methods, CFP and IWFNLP, are used to construct the fuzzy models. A heuristic MIHDE algorithm is then developed to solve the MINLP problems in Section 4. Section 5 summarizes the computational results obtained from applying the MIHDE algorithm to solve the crisp/fuzzy models. Concluding remarks are made in Section 6.

2. Problem descriptions and modelling

2.1. Problem descriptions

The customer demand of perishable goods is unpredictable or fuzzy in the real market. Lack of study on collaborative inventory model with fuzzy environment implies great opportunity for this type of research. Our study optimizes the total joint cost of a deteriorating item vendor–buyer replenishment model to consider fuzzy environment and constraints. Two different fuzzy decision-making approaches are used to formulate the fuzzy models in our analysis. The first method uses CFP and the second method uses IWFNLP; the latter is a heuristic algorithm with MIHDE. Numerical examples and sensitivity analysis are developed to investigate the effectiveness of the proposed method in the fuzzy environment. Subsequently, the optimum results derived from CFP and IWFNLP methods are compared with the existence method.

2.2. Mathematical modelling

A collaborative vendor–buyer ordering policy with inventory investment constraints is developed. The following notation is used:

p = annual vendor's production rate

d = annual consumer's demand rate

$I_{pb1}(t_1)$ = vendor's and buyer's total inventory level where the slope is positive

$I_{pb2}(t_2)$ = vendor's and buyer's total inventory level where the slope is negative

$I_p(t)$ = inventory level of the vendor

$I_b(t)$ = inventory level of the buyer

T_1 = time interval from $I_{pb1}(t_1) = 0$ to $I_{pb1}(t_1)$ = maximum inventory level

T_2 = time interval from $I_{pb2}(t_2)$ = maximum inventory level to $I_{pb2}(t_2) = 0$

T_a = time required by the vendor to produce the amount, δ

T_b = time required by the vendor to produce the first shipment lot size, Q

T = cycle time in integrated vendor–buyer system as illustrated in Figure 1

Q = buyer's lot size delivered from the vendor to the buyer

δ = the amount of initial stock required by the buyer during the first shipment production lot size, Q

n = number of deliveries from the vendor to the buyer per cycle time

θ = deterioration rate

C_{11} = setup cost for the vendor, $per production cycle

C_{21} = ordering cost for the buyer, $per order

C_{12} = inventory carrying cost for the vendor, $per unit time and per unit

C_{22} = inventory carrying cost for the buyer, $per unit time and per unit

C_{13} = unit production cost paid by the vendor, $per unit

C_{23} = unit purchase cost paid by the buyer, $per unit

AI_p = average inventory investment of the vendor

AI_b = average inventory investment of the buyer

MAI_p = maximum allowable average inventory investment for the vendor

MAI_b = maximum allowable average inventory investment for the buyer

TC_p = total cost for the vendor

TC_b = total cost for the buyer

TC = integrated joint cost including TC_p and TC_b

The supply chain inventory model can be depicted in Figure 1 below.

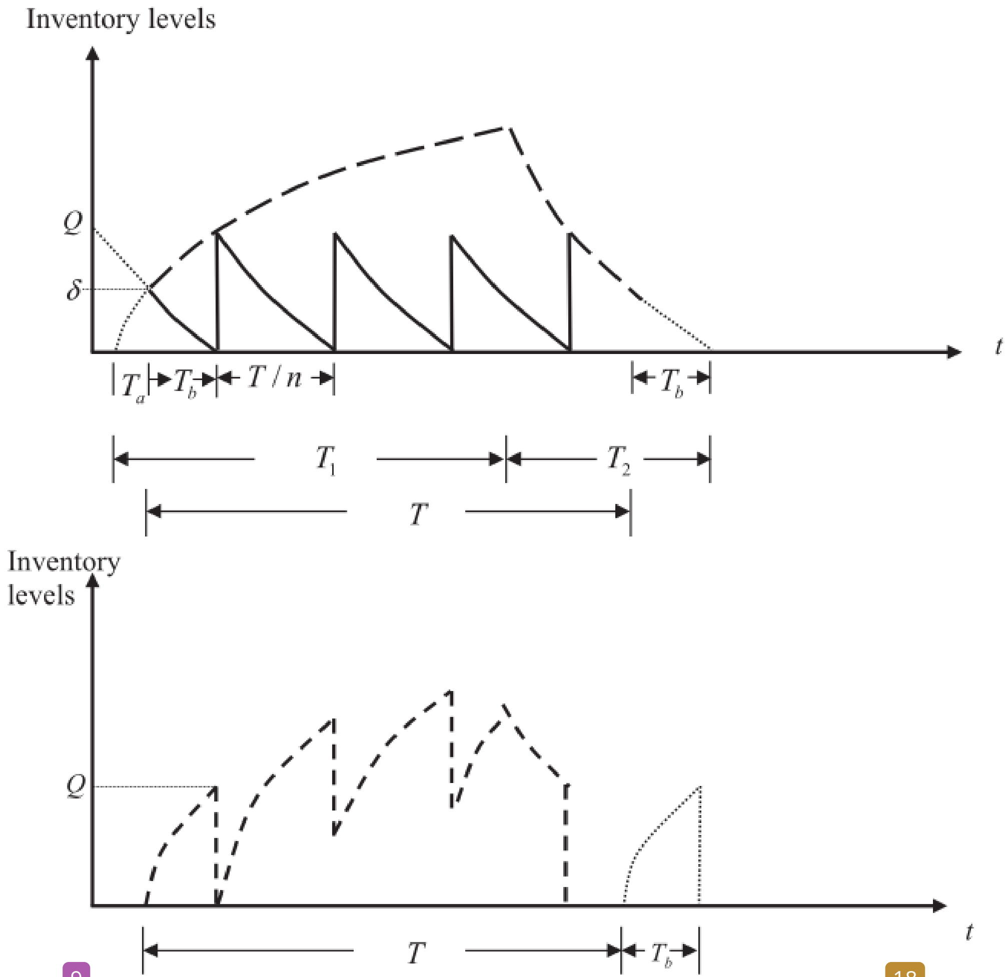

Figure 1. Vendor's and buyer's inventory levels for the case $n = 4$, where represents vendor's and buyer's total inventory level, $I_{pbi}(t_i)$, $i = 1$, 2 represents buyer's inventory level, $I_b(t)$ represents vendor's inventory level, $I_p(t)$.

In the collaborative vendor-buyer system, the buyer has n orders during the cycle time T. Inventory is depleted due to demand and deterioration. The differential equation governing the inventory status for the buyer is given by

$$\frac{dI_b(t)}{dt} = -d - \theta I_b(t) \qquad 0 \le t \le \frac{T}{n} \tag{1}$$

From the boundary condition $I_b(T/n) = 0$ and $I_b(0) = Q$, one has

$$I_b(t) = \frac{d}{\theta}\left(\exp\left(\theta\left(\frac{T}{n} - t\right)\right) - 1\right) \qquad 0 \le t \le \frac{T}{n} \tag{2}$$

and

$$Q = I_b(0) = \frac{d}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) \tag{3}$$

The average buyer's inventory level, I_b is

$$I_b = \frac{n}{T}\int_0^{T/n} I_b(t)dt = \frac{nd}{T\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right) \tag{4}$$

The loss of stock due to deterioration per cycle for the buyer is

$$n\left(Q - \frac{Td}{n}\right) \tag{5}$$

From Equations (4) and (5), the average cost per unit time for the buyer, TC_b, includes the ordering cost, the carrying cost and the deterioration cost. One has

$$TC_b = \frac{nC_{21}}{T} + \frac{C_{22}nd}{T\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right) + \frac{C_{23}n}{T}\left(\frac{d}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{Td}{n}\right) \tag{6}$$

From Equation (4), the average inventory investment for the buyer, AI_b is

$$AI_b = \frac{C_{22}nd}{T\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right) \tag{7}$$

Between 0 and T_1, the vendor's and the buyer's total inventory increase is the result of production. Between T_1 and $T_1 + T_2$, the total inventory decrease is the result of demand and deterioration. The differential equations governing the total inventory status is given by

$$\frac{dI_{pb1}(t_1)}{dt_1} = (p-d) - \theta I_{pb1}(t_1) \qquad 0 \le t_1 \le T_1 \tag{8}$$

$$\frac{dI_{pb2}(t_2)}{dt_2} = -d - \theta I_{pb2}(t_2) \qquad 0 \le t_2 \le T_2 \tag{9}$$

From the boundary condition $I_{pb1}(0) = I_{pb2}(T_2) = 0$, the differential equations are solved as follows:

$$I_{pb1}(t_1) = \frac{p-d}{\theta}(1 - \exp(-\theta t_1)) \qquad 0 \le t_1 \le T_1 \tag{10}$$

$$I_{pb2}(t_2) = -\frac{d}{\theta}\left(1 - \exp\left(\theta(T_2 - t_2)\right)\right) \qquad 0 \le t_2 \le T_2 \tag{11}$$

From the boundary condition $I_{pb1}(T_1) = I_{pb2}(0)$ and from Misra (1975), we have the following equation:

$$T_1 \approx \frac{d}{p-d} T_2\left(1 + \frac{1}{2}\theta T_2\right) \tag{12}$$

In Figure 1, δ is the amount of initial stock required by the buyer during the first shipment production lot size, Q. From Equation (10), we have

$$\delta = \frac{p-d}{\theta}\left(1 - \exp\left(-\theta T_a\right)\right) \tag{13}$$

We can also derive δ from Equation (2) as

$$\delta = \frac{d}{\theta}\left(\exp\left(\theta T_b\right) - 1\right) \tag{14}$$

where T_b is the time required by the vendor to produce the first shipment lot size, Q.

Between 0 and T_b, the differential equation governing the vendor's inventory status is given by

$$\frac{dI_p(t)}{dt} = p - \theta I_p(t) \qquad 0 \le t \le T_b \tag{15}$$

From the boundary condition $I_p(0) = 0$ and $I_p(T_b) = Q$, one can solve

$$I_p(t) = \frac{p}{\theta}\left(1 - \exp\left(-\theta t\right)\right) \qquad 0 \le t \le T_b \tag{16}$$

and

$$Q = \frac{p}{\theta}\left(1 - \exp\left(-\theta T_b\right)\right) \tag{17}$$

From Equations (3) and (17), one can derive

$$T_b = -\frac{1}{\theta}\ln\left(1 - \frac{d}{p}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right)\right) \tag{18}$$

From Equations (13), (14) and (18), and by Taylor's series approximations and the relation $ln(1-x) \approx -x$ when $x \ll 1$ and $\theta T/n \ll 1$, one can derive the followings

$$T_b \approx \frac{dT}{pn} \tag{19}$$

$$\delta \approx \frac{d^2 T}{pn} \tag{20}$$

and

$$T_a \approx \frac{d^2 T}{p(p-d)n} \tag{21}$$

respectively.
More detailed descriptions are shown in the Appendix.

In Figure 1, T is the cycle time for the integrated vendor–buyer system, where

$$T = T_1 + T_2 - T_a - T_b$$

From Equations (12), (19) and (21), one can derive

$$T = \frac{npT_2}{d^2 + d(p-d) + np(p-d)}\left(p + \frac{d\theta T_2}{2}\right) \tag{22}$$

$$T_a = \frac{d^2 T_2}{d^2(p-d) + d(p-d)^2 + np(p-d)^2}\left(p + \frac{d\theta T_2}{2}\right) \tag{23}$$

and

$$T_b = \frac{dT_2}{d^2 + d(p-d) + np(p-d)}\left(p + \frac{d\theta T_2}{2}\right) \tag{24}$$

The vendor's actual average inventory level, I_p, can be derived by subtracting the average buyer's inventory level, I_b, from the average vendor's and buyer's total inventory level, I_{pb} as follows

$$I_p = I_{pb} - I_b$$

$$\begin{aligned}
&= \frac{1}{T}\left[\int_{T_a}^{T_1} I_{pb1}(t_1)dt_1 + \int_{0}^{T_2-T_b} I_{pb2}(t_2)dt_2\right] - \frac{nd}{T\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right) \\
&= \frac{(p-d)}{T\theta}\left[(T_1 - T_a) + \frac{(\exp(-\theta T_1) - \exp(-\theta T_a))}{\theta}\right] \\
&\quad - \frac{d}{T\theta}\left[(T_2 - T_b) + \frac{(\exp(\theta T_b) - \exp(\theta T_2))}{\theta}\right] - \frac{nd}{T\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right)
\end{aligned} \tag{25}$$

The loss of stock due to deterioration per cycle for the vendor is

$$p(T_1 - T_a) - nQ \tag{26}$$

From Equations (25) and (26), the average cost per unit time for the vendor, TC_p, includes the setup cost, the carrying cost and the deterioration cost where

$$TC_p = \frac{C_{11}}{T} + \frac{C_{12}}{T}\left\{\frac{(p-d)}{\theta}\left[(T_1 - T_a) + \frac{(\exp(-\theta T_1) - \exp(-\theta T_a))}{\theta}\right]\right. \\ \left. -\frac{d}{\theta}\left[(T_2 - T_b) + \frac{(\exp(\theta T_b) - \exp(\theta T_2))}{\theta}\right] - \frac{nd}{\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right)\right\} \\ + \frac{C_{13}}{T}\left[p(T_1 - T_a) - \frac{nd}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right)\right] \tag{27}$$

From Equation (25), the average inventory investment for the vendor, AI_p is

$$AI_p = \frac{C_{12}}{T}\left\{\frac{(p-d)}{\theta}\left[(T_1 - T_a) + \frac{(\exp(-\theta T_1) - \exp(-\theta T_a))}{\theta}\right]\right. \\ \left. -\frac{d}{\theta}\left[(T_2 - T_b) + \frac{(\exp(\theta T_b) - \exp(\theta T_2))}{\theta}\right] - \frac{nd}{\theta}\left(\frac{1}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) - \frac{T}{n}\right)\right\} \tag{28}$$

The integrated joint cost TC for the vendor and the buyer is the sum of TC_p and TC_b. From Equations (22), (23) and (24), TC, AI_p and AI_b, are respectively showed in Equations (6)-(7), (27)-(28), can be written as a function of T_2 and n. Under the crisp environment, the integrated vendor–buyer inventory model with average inventory investment constraints of both vendor and buyer is formulated as

$$\begin{aligned} \text{Minimize} \quad & TC(n, T_2) = TC_b(n, T_2) + TC_p(n, T_2) \\ \text{subject to} \quad & AI_p(n, T_2) \le MAI_p, \\ & AI_b(n, T_2) \le MAI_b, \\ & T_2 \ge 0, \\ & n \ge 1, \text{where } n \text{ is an integer.} \end{aligned} \tag{29}$$

Due to fuzzy joint cost and constraints in the fuzzy environment, we assume non-rigid goals of the joint cost and constraints. The integrated model can be modelled as

$$\begin{aligned} \widetilde{\text{Minimize}} \quad & TC(n, T_2) \tilde{=} TC_b(n, T_2) + TC_p(n, T_2) \\ \text{subject to} \quad & AI_p(n, T_2) \tilde{\le} \widetilde{MAI}_p, \\ & AI_b(n, T_2) \tilde{\le} \widetilde{MAI}_b, \\ & T_2 \ge 0, \\ & n \ge 1, \text{where } n \text{ is an integer.} \end{aligned} \tag{30}$$

where the wavy bar '$\sim$' stands for fuzzification of the total cost and two constraints.

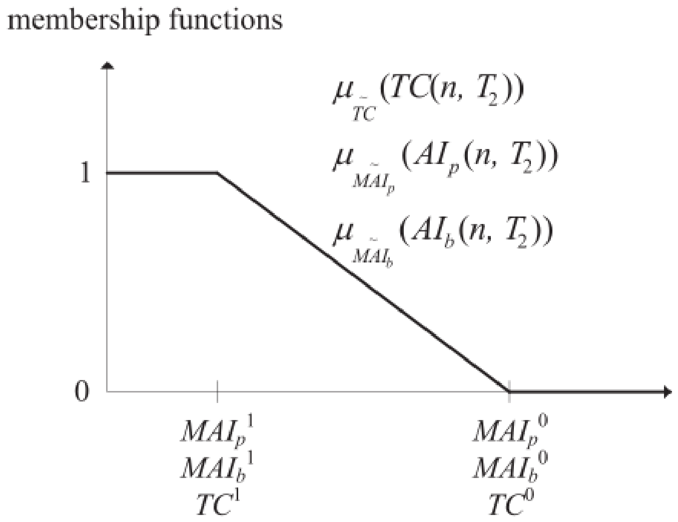

Figure 2. Membership functions for fuzzy total cost and two inventory investment constraints.

3. Two fuzzy decision-making approaches

Two fuzzy methods (CFP and IWFNLP) are used to construct the fuzzy joint cost models with fuzzy constraints given in Equation (30). The CFP method is based on the work by Tiwari et al. (1987); it maximizes the weighted sum of the membership functions. The second method using IWFNLP method embedded the idea of inverse weight proposed by Roy and Maiti (1998).

In the analysis, a system decision-maker may subjectively define the acceptable intervals for the joint cost $[TC^1, TC^0]$, the vendor's inventory investment $[MAI_p^1, MAI_p^0]$ and the buyer's inventory investment $[MAI_b^1, MAI_b^0]$. In fuzzy set theory, the fuzzy numbers $\tilde{TC}= [TC^1, TC^0]$, $\tilde{MAI}_p= [MAI_p^1, MAI_p^0]$ and $\tilde{MAI}_b= [MAI_b^1, MAI_b^0]$ are used to represent the fuzziness of the joint cost and two investment constraints. The membership functions, $\mu_{\tilde{TC}}(TC(n, T_2))$, $\mu_{\tilde{MAI}_p}(AI_p(n, T_2))$ and $\mu_{\tilde{MAI}_b}(AI_b(n, T_2))$ as calculated in Equations (31)–(33), respectively, represent the achievement degree of the objective and constraints in our study, where the membership functions are $\mu_A(x):U \to [0,\ 1]$.

Considering the cost model with the three constraints, the smaller relevant cost or inventory investment is preferred by the enterprise. For the acceptable intervals of the three constraints, our model uses linear decreasing numbers as described in the study of Pattnaik (2015) instead of the triangular fuzzy numbers. Hence, a simple fuzzy linear decreasing membership function representation is adopted and the pictorial representations of these functions are illustrated in Figure 2.

$$\mu_{\tilde{TC}}(TC(n, T_2)) = \begin{cases} 0 & \text{for } TC(n, T_2) > TC^0 \\ 1 - \dfrac{TC(n, T_2) - TC^1}{TC^0 - TC^1} & \text{for } TC^1 \le TC(n, T_2) \le TC^0 \\ 1 & \text{for } TC(n, T_2) < TC^1 \end{cases} \quad (31)$$

$$\mu_{\tilde{MAI}_p}(AI_p(n,T_2)) = \begin{cases} 0 & \text{for } AI_p(n,T_2) > MAI_p^0 \\ 1 - \dfrac{AI_p(n,T_2) - MAI_p^1}{MAI_p^0 - MAI_p^1} & \text{for } MAI_p^1 \le AI_p(n,T_2) \le MAI_p^0 \\ 1 & \text{for } AI_p(n,T_2) < MAI_p^1 \end{cases} \quad (32)$$

and

$$\mu_{\tilde{MAI}_b}(AI_b(n,T_2)) = \begin{cases} 0 & \text{for } AI_b(n,T_2) > MAI_b^0 \\ 1 - \dfrac{AI_b(n,T_2) - MAI_b^1}{MAI_b^0 - MAI_b^1} & \text{for } MAI_b^1 \le AI_b(n,T_2) \le MAI_b^0 \\ 1 & \text{for } AI_b(n,T_2) < MAI_b^1 \end{cases} \quad (33)$$

We apply CFP as our first method to establish the fuzzy model. The formulation is

$$\begin{aligned} &\text{Maximize } W_1\mu_{\tilde{TC}} + W_2\mu_{\tilde{MAI}_p} + W_3\mu_{\tilde{MAI}_b} \\ &\text{subject to } 1 - \frac{TC(n,T_2) - TC^1}{TC^0 - TC^1} = \mu_{\tilde{TC}}, \\ &1 - \frac{AI_p(n,T_2) - MAI_p^1}{MAI_p^0 - MAI_p^1} = \mu_{\tilde{MAI}_p}, \\ &1 - \frac{AI_b(n,T_2) - MAI_b^1}{MAI_b^0 - MAI_b^1} = \mu_{\tilde{MAI}_b}, \\ &T_2 \ge 0, n \ge 1, \text{where } n \text{ is an integer}, \\ &\mu_{\tilde{TC}}; \mu_{\tilde{MAI}_p}; \mu_{\tilde{MAI}_b} \in [0,1]. \end{aligned} \quad (34)$$

where W_1, W_2 and W_3 are positive weights provided by a decision-maker of an enterprise for the fuzzy joint cost and the fuzzy inventory investment constraints. The three transformed investment constraints including the total cost and vendor–buyer inventory cost are derived from their corresponding membership functions in Equations (31)–(33), respectively. Furthermore, these weights provided by a decision-maker of an enterprise expect to reflect explicitly the related importance of each desired target in a supply chain system. The CFP method can maximize the weighted sum of the membership functions, but the ratio of the achievement levels ($\mu_{\tilde{TC}}$, $\mu_{\tilde{MAI}_p}$ and $\mu_{\tilde{MAI}_b}$) obtained by this approach cannot reflect explicitly the ratio of the weights. The IWFNLP method is then proposed to overcome the drawback of the CFP method.

The ratio of the achievement levels should be close to the ratio of the weights to correspond to their related importance when the desired weights are provided. Therefore, we propose an IWFNLP method to construct the fuzzy model. For the weights W_j, $j = 1, \ldots, m$, one-to-one transformation from the weight W_j to the inverse weight IW_j is defined as follows:

$$IW_j = \frac{1/W_j}{\sum_{l=1}^{m} 1/W_l} = \frac{1}{W_j \sum_{l=1}^{m} 1/W_l} \tag{35}$$

Then the fuzzy model using the IWFNLP method is formulated as

$$\begin{aligned} &\text{Maximize } \lambda \\ &\text{subject to } IW_1\left(1 - \frac{TC(n,T_2) - TC^1}{TC^0 - TC^1}\right) \geq \lambda, \\ &IW_2\left(1 - \frac{AI_p(n,T_2) - MAI_p^1}{MAI_p^0 - MAI_p^1}\right) \geq \lambda, \\ &IW_3\left(1 - \frac{AI_b(n,T_2) - MAI_b^1}{MAI_b^0 - MAI_b^1}\right) \geq \lambda, \\ &T_2 \geq 0, n \geq 1, \text{where } n \text{ is an integer}, \\ &\lambda \geq 0. \end{aligned} \tag{36}$$

where $IW_j = 1/(W_j \sum_{l=1}^{3} 1/W_l)$, $j = 1, \ldots, 3$.

From the definition of the inverse weight, Equation (36) is equivalent to

$$\begin{aligned} &\text{Maximize } \lambda \\ &\text{subject to } 1 - \frac{TC(n,T_2) - TC^1}{TC^0 - TC^1} \geq W_1\lambda\left(\sum_{l=1}^{3} 1/W_l\right), \\ &1 - \frac{AI_p(n,T_2) - MAI_p^1}{MAI_p^0 - MAI_p^1} \geq W_2\lambda\left(\sum_{l=1}^{3} 1/W_l\right), \\ &1 - \frac{AI_b(n,T_2) - MAI_b^1}{MAI_b^0 - MAI_b^1} \geq W_3\lambda\left(\sum_{l=1}^{3} 1/W_l\right), \\ &T_2 \geq 0, n \geq 1, \text{where } n \text{ is an integer}, \\ &\lambda \geq 0. \end{aligned} \tag{37}$$

For the optimal solution obtained from Equation (37), the right sides of the constraints are identical to the left sides and have the same constant item, $\sum_{l=1}^{3} 1/W_l$. We have shown that the IWFNLP method can derive the optimal solution when the ratio of the achievement levels is close to the ratio of the weights. That means that the ratios of $\mu_{\tilde{TC}}/W_1$, $\mu_{M\tilde{A}I_p}/W_2$ and $\mu_{M\tilde{A}I_b}/W_3$ are all equivalent or nearly equivalent to λ.

4. Mixed-integer hybrid differential evolution

As the above formulated crisp/fuzzy models in Equations (29), (30), (34) and (36) or (37), these models are classified as mixed integer nonlinear programming (MINLP) problems. However, the MINLP problems are actually difficult to be optimized through mathematical programming, analytical method, or numerical analysis approach in a reasonable time. Hence, diversified and efficient heuristic algorithms are successively proposed to solve the MINLP problems by quite a lot researchers and scholars. A soft computing MIHDE problem solver engine is developed to solve the MINLP problems. MIHDE method is an improved hybrid differential evolution (HDE) method which is a simple population-based stochastic function optimization method proposed by Chiou and Wang (1999). Its main structure is to adopt the three different mechanisms in the multi-point linear search (i.e. mutation operation in HDE/DE method), reorganization (i.e. crossover operation in HDE/DE method), and selection to generate the next generation. Ten different linear mutation strategies in the parent individual and difference linear vector selection are suggested by Price et al. (2005) and Storn and Price (1996). Since the above models cannot be handled by HDE method because it uses a real coding to represent each decision variable (i.e. gene in HDE represents decision variable in the real-world problem), MIHDE method is used to derive the optimal solution.

In the MIHDE method, a rounding operator is embedded into HDE to obtain the nearest integer value for each integer decision variable (gene). Finally, the evaluation of fitness function is used to improve the MIHDE method which is proposed by Deb (2000) who developed an efficient constraint handing method for genetic algorithm.

The MINLP problem is formulated as follows

$$\begin{aligned} \underset{\mathbf{Z}=(\mathbf{x},\ \mathbf{y})}{\text{Minimize}}\ & f(\mathbf{x},\mathbf{y}) \\ \text{subject to}\ & g_j(\mathbf{x},\mathbf{y}) = 0,\ \ j = 1,\ldots,p, \\ & g_j(\mathbf{x},\mathbf{y}) \le 0,\ \ j = p+1,\ldots,q, \\ & \mathbf{x}^L \le \mathbf{x} \le \mathbf{x}^U, \\ & \mathbf{y}^L \le \mathbf{y} \le \mathbf{y}^U. \end{aligned} \tag{38}$$

where $\mathbf{x} = (x_1, \ldots, x_d)$ is a d-dimensional vector of continuous decision variables, $\mathbf{y} = (y_{d+1}, \ldots, y_n)$ is an $(n - d)$-dimensional vector of discrete decision variables, $h_j(\mathbf{x}, \mathbf{y}), j = 1, \ldots, p$, are p equality constraints, $g_j(\mathbf{x}, \mathbf{y}), j = p+1, \ldots, q$, are $(q - p)$ inequality constraints. The $(\mathbf{x}^L, \mathbf{y}^L)$ and $(\mathbf{x}^U, \mathbf{y}^U)$ are the lower and upper bounds of the corresponding decision variables. (Note that p should be less than n. The optimal MINLP problem becomes a root-finding problem or problem solving equations if $p = n$ and may have no feasible solution if $p > n$).

The basic structure of MIHDE is shown in Table 2. The structure is a parallel direct search algorithm that utilizes Np vectors for decision variables $\mathbf{Z} = (\mathbf{x}, \mathbf{y})$ as population in generation G.

Next, we briefly illustrate the procedure of the MIHDE heuristic algorithm. In Step 1, the initial population is randomly selected from the entire search space as shown in the following form:

$$(\mathbf{x}^0, \mathbf{y}^0)_i = (\mathbf{x}^L, \mathbf{y}^L) + \left\{\rho_i(\mathbf{x}^U - \mathbf{x}^L),\ \text{INT}\left[\rho_i\left(\mathbf{y}^U - \mathbf{y}^L\right)\right]\right\}\ i = 1,\ \ldots, N_p \tag{39}$$

where $\rho_i \in U(0, 1)$. The operator $\text{INT}[b]$ is to find the nearest integer vector to the real vector b.

Table 2. Basic steps and procedure of MIHDE.

Step	Procedure
1	Develop a mixed-coding representation and initialize all parameters for MIHDE
2	Update each individual by mutation operation
3	Update each individual by crossover operation
4	Select and evaluate the best individual by calculating the fitness functions of each individual
5	Adopt acceleration operation when the best solution is not continuously improved
6	Adopt migration operation to generate a new population when the current solutions prematurely cluster around the best individual
7	Repeatedly execute Steps 2 to 6 until the stopping criterion is satisfied

The mutation operation in MIHDE uses the difference in the vector between two random chosen individuals as a search direction, as shown in the right-hand side of the equality constraint in Equation (40) in Step 2. On the basis of the parent individual $(\mathbf{x}^G, \mathbf{y}^G)_p$, the i^{th} temporary mutant individual $(\hat{\mathbf{x}}^G, \hat{\mathbf{y}}^G)_i$ is generated by the difference in the vector between two random chosen individuals at G^{th} iteration, one has

$$\left(\left(\hat{\mathbf{x}}^G, \hat{\mathbf{y}}^G\right)_i = \left(\mathbf{x}^G, \mathbf{y}^G\right)_p + \left\{\mathrm{F}\left(\mathbf{x}_u^G - \mathbf{x}_v^G\right), \mathrm{INT}\left[\mathrm{F}\left(\mathbf{y}_u^G - \mathbf{y}_v^G\right)\right]\right\} \quad i = 1, \ldots, N_p \qquad (40)$$

where u, v, $u \neq v$, are randomly generated from (1, Np), and F is a mutation factor.

Price et al. (2005) and Storn and Price (1996) introduced the interval (0, 1.2] for F to ensure the quickest possible convergence. As shown in Equation (40), the temporary mutant individual, $(\hat{\mathbf{x}}^G, \hat{\mathbf{y}}^G)_i$, $i = 1, \ldots, Np$, is basically a perturbed duplicate of the parent individual $(\mathbf{x}^G, \mathbf{y}^G)_p$ at G^{th} iteration. Therefore, the parent individual $(\mathbf{x}^G, \mathbf{y}^G)_p$ selected is based on the type of mutation operations. Different from GA, the mutation operation with the parent individual and linear vector selection (MIHDE method) implemented in each iteration/generation is to enhance the diversified search in the process of evolution.

For improving the diversity of the new population at the next iteration, a binomial crossover procedure is to generate all offspring of the population at G^{th} iteration in Step 3. In this crossover operation, k^{th} real value gene and l^{th} discrete value gene for the i^{th} individual at the $(G+1)^{th}$ generation is produced from the temporary mutation individual $(\hat{\mathbf{x}}^G, \hat{\mathbf{y}}^G)_i$ and the current individual $(\mathbf{x}^G, \mathbf{y}^G)_i$; one has:

$$\dot{x}_{ki}^{G+1} = \begin{cases} x_{ki}^G & \text{if a random number} > C_R \\ \hat{x}_{ki}^G & \text{otherwise}; \quad k = 1, \ldots, d; \quad i = 1, \ldots, N_p \end{cases} \qquad (41)$$

and

$$\dot{y}_{li}^{G+1} = \begin{cases} y_{li}^G & \text{if a random number} > C_R \\ \hat{y}_{li}^G & \text{otherwise}; \quad l = d+1, \ldots, n; \quad i = 1, \ldots, N_p \end{cases} \qquad (42)$$

where the crossover factor $C_R \in [0, 1]$ is set by the user-specified.

Two selection steps are proposed in this evaluation operation in Step 4. The first selection step is the one-to-one competition for obtaining the better individual from either the parent or its offspring. If the offspring $(\dot{\mathbf{x}}^{G+}1, \dot{\mathbf{y}}^{G+}1)_i$ obtained in the crossover operation survives, the fitness function value of the offspring is superior to its parent and it is the minimization; otherwise the parent is retained. In MIHDE, the fitness function introduced by Deb (2000) is obtained by considering a penalty term independent of a penalty parameter. The fitness function of an individual is

$$\text{For } j=1,\ 2,\ \ldots,\ q,$$
$$F(\mathbf{x},\mathbf{y}) = \begin{cases} f(\mathbf{x},\mathbf{y}) & \text{if } g_j(\mathbf{x},\mathbf{y}) \le 0 \text{ and } g_j(\mathbf{x},\mathbf{y}) = 0 \\ f_{\max} + \left\{ \sum_{j=p+1}^{q} \left(\max[0, g_j(\mathbf{x},\mathbf{y})]\right) + \sum_{j=1}^{p} |g_j(\mathbf{x},\mathbf{y})| \right\} & \text{otherwise;} \end{cases} \tag{43}$$

where $f_{\max}$ is the maximum objective function value for all feasible solutions in the population.

Each individual $(\mathbf{x}^{G+1}, \mathbf{y}^{G+1})_i$ in the population is then derived based on their objective function values with and without constraints. The best individual $(\hat{\mathbf{x}}^{G+}1,\ \hat{\mathbf{y}}^{G+}1)_b$ in the population is then determined from the next selection step. They are expressed as follows

$$(\mathbf{x}^{G+1}, \mathbf{y}^{G+1})_i = \operatorname{argmin}\left\{ f(\mathbf{x}^G, \mathbf{y}^G)_i, f(\dot{\mathbf{x}}^{G+1}, \dot{\mathbf{y}}^{G+1})_i \right\},\ i = 1,\ \ldots,\ N_p \tag{44}$$

$$\left(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1}\right)_b = \operatorname{argmin}\left\{ f\left(\mathbf{x}^{G+1}, \mathbf{y}^{G+1}\right)_i \right\},\ i = 1,\ \ldots,\ N_p \tag{45}$$

In MIHDE, an acceleration and migration operations are embedded in the original differential evolution (DE). The two key operations in MIHDE are able to reduce a premature convergence and to explore the diversity from the entire search space and the two aspects is regarded as trade-off operators between convergence and diversification. To improve the acceleration operation, a descend method is applied to derive at the best solution. The acceleration operation in Step 5 is expressed as

$$\left(\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1}\right)_b = \begin{cases} \left(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1}\right)_b & \text{if } f\left(\left(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1}\right)_b\right) < f\left(\left(\mathbf{x}^{G}, \mathbf{y}^{G}\right)_b\right) \\ \left(\hat{\mathbf{x}}_b^{G+1} - \alpha \nabla f_{\mathbf{x}}\left(\left(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1}\right)_b\right),\ INT\left[\hat{\mathbf{y}}_b^{G+1} - \alpha \nabla f_{\mathbf{y}}\left(\left(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1}\right)_b\right)\right]\right) & \text{otherwise} \end{cases} \tag{46}$$

where $(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1})_b$ denotes the temporary best individual at the $(G+1)^{th}$ generation from Equation (45), $(\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1})_b$ denotes the current best individual at the $(G+1)^{th}$ generation from Equation (46), $\nabla f((\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1})_b)$ is the gradient of the objective function at $(\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1})_b$, and $\alpha \in (0, 1]$ is the step size. It is set to one initially and then reduced the step size until the new individual with minimizing the fitness function value is found. To obtain a better individual $(\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1})_b$ at $(G+1)^{th}$ generation such that one is to satisfy with $f((\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1})_b) < f((\mathbf{x}^{G}, \mathbf{y}^{G})_b)$, the descend method is repeated using a smaller step size until $\alpha \nabla f((\hat{\mathbf{x}}^{G+1}, \hat{\mathbf{y}}^{G+1})_b)$ is sufficiently small, or an iteration limit is exceeded. When a better individual $(\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1})_b$ is found, the solution $(\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1})_b$ becomes a candidate and is regarded as the best solution $(\mathbf{x}^{G+1}, \mathbf{y}^{G+1})_b = (\bar{\mathbf{x}}^{G+1}, \bar{\mathbf{y}}^{G+1})_b$ in the next iteration. Finally, the current best solution $(\mathbf{x}^{G+1}, \mathbf{y}^{G+1})_b$ replaces the worst individual.

Although the acceleration operation can improve the convergence rate, the candidate may gradually cluster around the best individual. As shown in Equations (40)–(42), these clustered individuals cannot reproduce a better individual through the mutation and crossover operation, and yield a premature convergence or a local optimum. Therefore, a migration operation in MIHDE is performed to avoid this local cluster in Step 6. The migration operation regenerates a new population based on the current best individual$(\mathbf{x}^{G+1}, \mathbf{y}^{G+1})_b$; one has

$$(\mathbf{x}^{G+1}, \mathbf{y}^{G+1})_i = \left(\mathbf{x}_b^{G+1} + N_1(0, \quad \sigma),\ INT\left[\mathbf{y}_b^{G+1} + N_2(0,\sigma)\right]\right)\ i = 1,\ \ldots,\ Np, \qquad i \neq b \tag{47}$$

where $N_1(0, \quad \sigma)$ and $N_2(0, \quad \sigma)$ are the vectors of independent random Gaussian numbers with mean 0 and standard deviation σ. We perform the migration operation when the degree of population diversity failed to satisfy a desired tolerance. There are two steps process in measuring the population diversity.

Firstly, with respect to the best individual, define k^{th} real value gene or l^{th} discrete value gene of the i^{th} individual as a diversified gene, if

$$\left|\frac{x_{ki}^{G+1} - x_{kb}^{G+1}}{x_{kb}^{G+1}}\right| > \varepsilon_2 \text{ or } y_{li}^{G+1} \neq y_{lb}^{G+1} \tag{48}$$

is satisfied, the x_{kb}^{G+1} and y_{lb}^{G+1} values are, respectively, k^{th} real value gene and l^{th} discrete value gene of the best individual in the population. The ε_2 value is the assigned gene diversity tolerance. The index η_{ki}=1 or η_{li}=1 is used to represent the diversified degree of real value or discrete value gene when the diversified gene of the x_{kb}^{G+1} or y_{lb}^{G+1} is corresponding to the above condition of Equation (48). If the above condition fails, the clustered gene has η_{ki}=0 or η_{li}=0. The second procedure applied η_{ki} and η_{li} to measure the degree of population diversity ρ, defined as

$$\rho = \frac{\left(\sum_{\substack{i=1\\ i\neq b}}^{N_p}\left(\sum_{k=1}^{d}\eta_{ki} + \sum_{l=1}^{n-d}\eta_{li}\right)\right)}{(n)(N_p - 1)} \tag{49}$$

If ρ is less than the desired tolerance perform migration operation to regenerate a new population. From Equation (49), the degree of population diversity is between zero and one. Finally, execute Steps 2 to 6 repeatedly until the stopping criterion is satisfied.

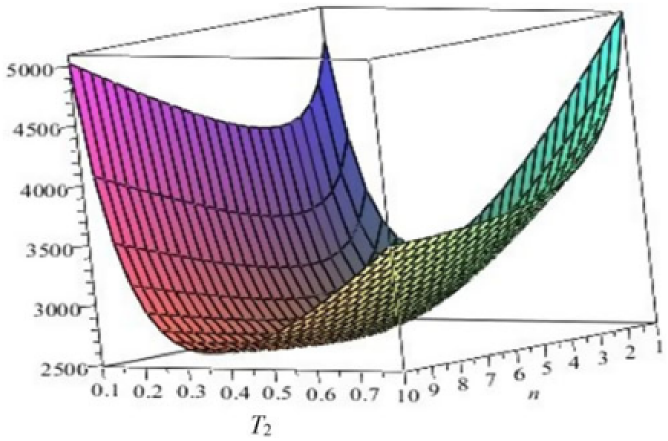

Figure 3. Graphic illustration for the integrated ordering policy for the unconstrained crisp model.

5. Numerical example

The data from Goyal (1988) with unconstrained and non-deteriorating item are used for numerical illustration. The parameter values are: C_{11} = \$400/setup, C_{12} = \$4/unit/year, $p = 3200$ units/year, C_{21} = \$25/order, C_{22} = \$5/unit/year and $d = 1000$ units/year. For the crisp and fuzzy deteriorating item integrated models, we take $\theta = 0.1$, C_{13} = \$40/unit, C_{23} = \$50/unit, $MAI_b{}^1$ = \$1000/year, $MAI_b{}^0$ = \$1400/year, $MAI_p{}^1$ = \$3000/year and $MAI_p{}^0$ = \$4800/year. The crisp models with/without inventory investment constraints and fuzzy are solved using CFP, IWFNLP and MIHDE methods. The setting factors for MIHDE are listed as follows. The population size $Np = 10$, crossover factor $CR = 0.5$, mutation factor $F = 0.3$, tolerance of population diversity $\varepsilon_1 = 0.01$, and tolerance of gene diversity $\varepsilon_2 = 0.01$. The maximum generation is set to be 2500 for all inventory replenishment models including the crisp/fuzzy models; the fuzzy models consider two fuzzy methods (CFP and IWFNLP methods).

Assuming the vendor and the buyer have unlimited resources, we first develop an integrated ordering policy for the unconstrained crisp model and demonstrate that the graphic illustration of the unconstrained crisp model can be illustrated as shown in Figure 3. Due to the complication of this model, we then develop the MIHDE method to obtain the optimization of this model with/without fuzzy constraint environments. The optimal solution with the MIHDE method on the unconstrained crisp model are n^* = 5 and $T_2{}^*$= 0.3150. The total vendor and buyer costs are TC_p (n, T_2) = \$1769.1 and TC_b (n, T_2) = \$719.4, respectively. The joint cost of the integrated system is TC (n, T_2) = \$2,488.5. In order to verify the correctness and validity of the MIHDE method, we, respectively, show that the pattern plots with fixing one of the decision variables to determine the trend curve of the remainder are shown in Figure 4a and b. Comparison with Figures 3, 4a and b and the MIHDE method, the optimal solution and cost are the same results. Then, substituting n^* and $T_2{}^*$ into Equation (22), we can derive the optimal cycle time T^* = 0.4220. Substituting n^* and T^* into Equation (3), we derive the optimal buyer's integrated policy with lot size, Q^*= 85 units.

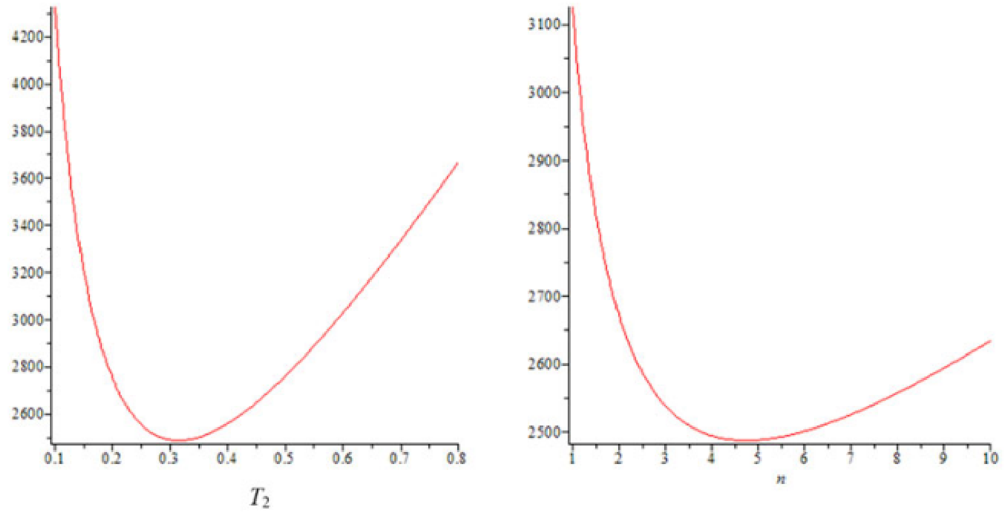

Figure 4. (a) the pattern plot with fixing, $n^* = 5$ the decision variable, (b) the pattern plot with fixing the decision variable, $T_2{}^* = 0.3150$.

If the vendor and the buyer determine their inventory policies independently, for $n = 1$, the buyer can compute her economic order cycle time and order quantity using Equations (6) and (3), respectively. The buyer's order cycle time is 0.0705 with $Q = 71$ units; the total annual cost TC_b is \$707.9. The vendor uses Equation (27) to determine n (i.e. the number of delivery orders) per cycle with T/n equal to buyer's order cycle time. The optimal number of deliveries for the vendor is $n = 6$ with a total annual cost TC_p = \$1794.8. The joint cost for the non-integrated model is \$2502.7. The joint cost for an integrated model is \$2488.5. Therefore, the cost of the integrated model is \$14.2 lower.

In order to enhance cooperation, a cost sharing mechanism of the integrated model is suggested by Goyal (1977). It is to enable each member to enjoy the benefit of integration. The total annual cost saving of an integrated model is shared by the vendor and the buyer, the cost sharing weight is

$$\omega = \frac{TC_b}{TC_b + TC_p}$$

Cost to the retailer $= \omega TC(n, T_2)$
Cost to the manufacturer $= (1 - \omega) TC(n, T_2)$

In our numerical example, ω is $707.9/(707.9 + 1794.8) = 0.2829$. The allocated buyer's total annual cost is \$2488.5 × 0.2829 = 704.0 and the allocated vendor's total annual cost is \$2488.5 × (1−0.2829) = 1784.5. The resulting allocation is summarized in Table 3. In order to encourage the buyer to cooperate with him, the vendor should compensate the buyer \$719.4 − \$704.0 = \$15.4 per year.

For the constrained crisp model, we use parameter settings $MAI_b{}^1$ = \$1000/year, $MAI_b{}^0$ = \$1400/year, $MAI_p{}^1$ = \$3000/year, and $MAI_p{}^0$ = \$4800/year to construct four different planning scenarios. To elicit the acceptable interval of the integrated joint cost, the joint cost optimization under each constraint scenario is independently

Table 3. Allocation of the total annual cost.

Non-integrated model		Integrated model	
Buyer's order quantity	71	Buyer's order quantity	85
Number of delivery orders	6	Number of delivery orders	5
Cycle time	0.4230	Cycle time	0.4220
Buyer's total annual cost	$707.9	Buyer's total annual cost	$719.4
		Allocated buyer's annual cost	$704.0
Vendor's total annual cost	$1794.8	Vendor's total annual cost	$1769.1
		Allocated vendor's annual cost	$1784.5

Table 4. Optimal results of crisp model.

	Average inventory investment constraints					
Scenarios	MAI_p	MAI_b	n	T_2	T	TC
1	$MAI_p^1 = \$3000$	$MAI_b^1 = \$1000$	6	0.1764	0.2391	$2980
2	$MAI_p^1 = \$3000$	$MAI_b^0 = \$1400$	5	0.1824	0.2439	$2872
3	$MAI_p^0 = \$4800$	$MAI_b^1 = \$1000$	9	0.2586	0.3595	$2697
4	$MAI_p^0 = \$4800$	$MAI_b^0 = \$1400$	7	0.2744	0.3764	$2569

performed. In Table 4, from scenario 1 to 4, we show that the larger the inventory investment for the vendor or the buyer, the smaller the integrated joint cost. However, the amount of inventory investment is controlled by the decision maker of the integrated supply chain. Hence, a compromise between the optimal integrated joint cost and inventory investment constraints should be achieved.

To illustrate the fuzzy models, we assume a fluctuation in the projected vendor's and buyer's average inventory investment where $\tilde{MAI}_p = [3000, 4800]$ and $\tilde{MAI}_b = [1000, 1400]$. From the computational results of the constrained crisp model in Table 4, $\tilde{TC} = [2569, 2980]$. The members functions of $\tilde{TC}$, $\tilde{MAI}_p$, and $\tilde{MAI}_b$are given in Equations (31), (32) and (33), respectively. The other data are described in the first paragraph of Section 5. With the relative importance of the fuzzy joint cost, fuzzy vendor's inventory investment constraint and buyer's inventory investment constraint, the respective weights W_1, W_2 and W_3 are assigned by the decision-maker. The optimal results derived from the CFP and IWFNLP methods are presented in Table 5.

Table 5 shows four types of fuzzy joint cost and two fuzzy constraints. In Type A, the three components have the same importance and equal weights. In Type B, more importance has been given to the fuzzy joint cost. Similarly, Type C and Type D, respectively, focused more on the fuzzy vendor's and buyer's inventory investment constraints. From Type B to D, the weight of 0.48 is assigned to the maximum fuzzy component.

From Table 5, the CFP method with fuzzy joint cost and constraints maximizes the weighted sum of each achievement level shown in the last column. As shown in Type B, the achievement levels of both the fuzzy joint cost and vendor's constraint are lower than that of the fuzzy buyer's constraint. The fuzzy buyer's constraint has the lowest weight among the three fuzzy components. Similarly, in Type C, the achievement level of the fuzzy joint cost is only 0.010; this is far less than its weight, $W_1 = 0.2$. It seems that this method cannot fully reflect the expectation of the decision-maker. This is because the decision-maker wants to achieve a higher benefit

Table 5. Optimal results of fuzzy model.

Method	TC	$\mu_{\tilde{T}C}$	$\frac{\mu_{\tilde{T}C}}{W_1}$	AI_p	$\mu_{M\tilde{A}I_p}$	$\frac{\mu_{M\tilde{A}I_p}}{W_2}$	AI_b	$\mu_{M\tilde{A}I_b}$	$\frac{\mu_{M\tilde{A}I_b}}{W_3}$	$\sum W\mu$
Type A: Equal importance to fuzzy joint cost and two fuzzy constraints. $W_1 = W_2 = W_3 = 1/3$.										
IWFNLP	\$2752	0.556	1.668	\$3658	0.634	1.902	\$1177	0.558	1.674	0.583
CFP	\$2830	0.366	1.098	\$3560	0.689	2.067	\$1000	1.000	3.000	0.685
Type B: More importance to fuzzy joint cost. $W_1 = 0.48$; $W_2 = 0.32$; $W_3 = 0.2$										
IWFNLP	\$2670	0.756	1.575	\$3833	0.537	1.678	\$1274	0.315	1.575	0.598
CFP	\$2734	0.600	1.250	\$3867	0.518	1.619	\$1086	0.785	3.925	0.611
Type C: More importance to fuzzy vendor's inventory investment constraint. $W_1 = 0.2$; $W_2 = 0.48$; $W_3 = 0.32$										
IWFNLP	\$2838	0.346	1.730	\$3304	0.831	1.731	\$1132	0.670	2.094	0.682
CFP	\$2976	0.010	0.050	\$3008	0.996	2.075	\$1000	1.000	3.125	0.800
Type D: More importance to fuzzy buyer's inventory investment constraint. $W_1 = 0.32$; $W_2 = 0.2$; $W_3 = 0.48$										
IWFNLP	\$2738	0.590	1.844	\$4137	0.368	1.840	\$1006	0.985	2.052	0.735
CFP	\$2744	0.576	1.766	\$4114	0.381	1.905	\$1000	1.000	2.083	0.741

$\sum W\mu$ is the weighted sum $W_1\mu_{\tilde{T}C} + W_1\mu_{M\tilde{A}I_p} + W_1\mu_{M\tilde{A}I_b}$ of each fuzzy component.

than other intuitions by designing a higher weight fuzzy joint cost. Therefore, our study applies the IWFNLP method to overcome the drawbacks of the CFP method. On the other hand, when the IWFNLP method is used, the ratios of the achievement levels to the weights for the fuzzy joint cost, fuzzy vendor's and buyer's inventory investment constraints are nearly identical. The results are significant. For example, in Type B, the fuzzy joint cost, fuzzy vendor's and buyer's inventory investment constraints are 0.756/0.48 = 1.575, 0.537/0.32 = 1.678 and 0.315/0.2 = 1.575, respectively. When the difference between the ratios of the three components is small, the IWFNLP method can obtain a minimal total cost using the MIHDE method. The optimum solutions and results derived by the IWFNLP method can fulfill the management's desirable achievement level. Hence, the IWFNLP method is an efficient decision tool for solving the fuzzy model with fuzzy environment and resource constraints.

6. Conclusions and future research

The fuzzy model has been formulated using the convex fuzzy programming (CFP) and inverse weighted fuzzy non-linear programming (IWFNLP) methods. These models are classified as MINLP problems. To solve the NP-hard problem within a reasonable computational time, a mixed integer hybrid differential evolution (MIHDE) technique was used in our study. The MIHDE method has proved to be effective computational tool in searching for good solutions, and is popular among managers in making decisions under real-life marketing environments. In this study, we have demonstrated that the CFP method has maximized the weighted sum of each achievement level. Moreover, our results have shown that the IWFNLP method is a more efficient decision tool; the result of our study conforms to the expectation of the system operators and management. For future research, permissible backlogging, delay in payments and for multiple vendors and buyers supply chain can be considered.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix

For $\theta T/n \ll 1$, $\exp(\theta T/n)$ is replaced by $1 + \frac{\theta T}{n} + \frac{1}{2!}\left(\frac{\theta T}{n}\right)^2$.

The percentage error for the third term in the Taylor series is

$$\frac{\frac{1}{2!}\left(\frac{\theta T}{n}\right)^2}{1 + \frac{\theta T}{n} + \frac{1}{2!}\left(\frac{\theta T}{n}\right)^2}$$

When $\theta T/n \leq 0.03$, the percentage error is about 0.04%. It will be smaller for term higher than three. Therefore, the term three and higher are neglected.

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